

UNIVERSITY OF OSLO  
Faculty of Mathematics and Natural Sciences

Midterm Exam AST2110 — The Universe

Date of exam: Monday 14th March 2005

Time for exam: 13.30 – 16.30

This exercise set contains 3 pages.

Attachments: None

Allowed sources of help: Rottmann: “Matematisk formel-samling”

Øgrim og Lian: “Størrelser og enheter i fysikk og teknikk”

Calculator

Two A4 pages (can be written on both sides) with your own notes

*Check that the exercise set is complete before you start solving the problems.*

Oppgave 1

A satellite orbiting the Earth has an orbit with semimajor axis  $a = 10000$  km and eccentricity  $e = 0.2$ . The Earth is assumed to be spherical with radius 6378 km and mass  $5.974 \times 10^{24}$  kg. The gravitational constant  $G = 6.6726 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$ .

- a) Calculate the orbital period of the satellite.
- b) Calculate the smallest and the largest altitude of the satellite above the surface of the Earth.
- c) Calculate the smallest and the largest velocity of the satellite in its orbit.

## Exercise 2

- a) We send a particle with mass (rest mass)  $m_0$  toward a target particle with the same mass. We assume that the target particle is free and at rest in the laboratory frame of reference. Let  $E_L$  denote the total energy (kinetic and rest energy) of both the two particles in the laboratory frame, and let  $E_C$  denote the total energy in the centre of mass frame. Find  $E_C$  expressed in terms of  $E_L$ .
- b) Anti-protons have been produced by sending a proton beam toward a target of protons. When a proton hits another proton, the result may be three protons and an anti-proton,  $P + P \rightarrow P + P + P + \bar{P}$ . In the following, we disregard the binding energy of the anti-proton. With threshold energy for this process is meant the kinetic energy (in the laboratory frame) that the incoming proton at least must have for this process to be possible. Find the threshold energy.
- c) Using storage rings, one operates with colliding beams of particles. Assume that we have two proton beams with the

same energy and opposite directions. Find the threshold energy for the process in this case.

### Oppgave 3

The Planck spectrum ( $F_\nu = \pi B_\nu$ ) is given by

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}. \quad (1)$$

- a) Explain the quantities in this equation. Express the Planck spectrum by wavelength instead of frequency,  $B_\lambda(T)$ .
- b) Find how the Planck spectrum behaves at very long wavelengths,  $hc/\lambda \ll kT$  (Rayleigh-Jeans' law).
- c) What is the total bolometric flux from a black body? [ $\int_0^\infty \frac{x^3 dx}{e^x - 1} = \pi^4/15$ ]